On the Equivalence Principle and gravitational and inertial mass relation of classical charged particles

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Abstract

We show that the locally constant force necessary to get a stable hyperbolic motion regime for classical charged point particles, actually, is a combination of an applied external force and of the electromagnetic radiation reaction force. It implies, as the strong Equivalence Principle is valid, that the passive gravitational mass of a charged point particle should be slight greater than its inertial mass. An interesting new feature that emerges from the unexpected behavior of the gravitational and inertial mass relation, for classical charged particles, at very strong gravitational field, is the existence of a critical, particle dependent, gravitational field value that signs the validity domain of the strong Equivalence Principle. For electron and proton, these critical field values are $g_c \simeq 4.8 \times 10^{31} m/s^2$ and $g_c \simeq 8.8 \times 10^{34} m/s^2$, respectively.

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1 Introduction

The problem of the electromagnetic radiation reaction force on the charged particle dynamics, as given by the Lorentz-Abraham-Dirac (LAD) equation [1]-[11], has been a subject of active investigation. There are a lot of works about this subject accumulated since the first attempt was made by Dirac [1].

There is now a renewed interest on this subject, with works pointing to something new, which should affect the validity of the Weak Equivalence Principle at some circumstances [12]-[21]. Perhaps because the main experimental justification that led Einstein to formulate the Equivalence Principle (EP), which is one of the foundations of his General Theory of Relativity, is the numerical equality between inertial and gravitational mass, nowadays they are taken quite as synonymous, so we have to be aware to avoid misleading conclusions. According to Weinberg [5], we distinguish the Weak Equivalence Principle (WEP) of the Strong Equivalence Principle (SEP). The Strong Equivalence Principle postulates that at every space-time point in a arbitrary gravitational field it is possible to choose a locally inertial coordinate system such that, within a sufficiently small region of the point in question, the laws of the nature take the same form as in unaccelerated Cartesian coordinate systems in the absence of gravitation. On the other hand, the Weak Equivalence Principle is nothing but a restatement of the observed equality of gravitational and inertial mass.

About the verification of the WEP, there is a surprising richness in the variety of experimental techniques and choice of the test bodies which have been used so far. The equality of gravitational and inertial mass is in fact what the experiments, since the famous Eötvos balance until recents experiments, actually measure. We show a brief review. The most obvious way to proof the WEP is to compare the motion of two bodies during free fall. These experiments, limited by rather short free falling periods, reached an accuracy of about 1 part in 10^{-10} [22]. The Bremen drop tower experiments, using SQUID displacement sensors, will provide a much longer time for free fall, allowing to reach an accuracy of about $10^{-12} - 10^{-13}$ [23, 24]. Torsion balance experiments have reached an accuracy of few parts in 10^{-13} [25]. A planned experiment using a cryogenic balance claims an accuracy of 10^{-14} [26]. The most sensitive long-range measurements have used the Sun as the source and Earth and Moon type test bodies. The lunar laser ranging (LLR) techniques reach an accuracy of 5×10^{-13} [27, 28]. On the other hand, future space experiments promise much better precision in this measurement. The MICROSCOPE mission [29] aims to test, on a microsatellite of the MYRIADE series developed by CNES/FRANCE, the WEP with a 10^{-15} accuracy. The Galileo Galilei-GG is a proposed experiment in low orbit around the Earth aiming to test the WEP to the level of 1 part in 10^{-17} [30]. STEP, using pairs of concentric free-failing proof-masses, will be able to test the WEP to a sensitivity at 1 part in 10^{-18} [31].

On the other hand, notice that these mentioned experiments don't investigate the WEP in the case of charged particles. The reason is that electromag-

netic fields influence gravitation experiments with charged particles and must be shielded carefully. The experiments for freely falling electrons carried out by Witteborn and Fairbank [32], with an accuracy of 10^{-1}, is the only one cited in literature. Nowadays, Dittus and Lämmerzahl [33] showed that an experiment in space with the Witteborn-Fairbank set-up may be well suited to test the WEP and to improve the results for free fall test with charged particles by orders of magnitude.

Some important comments should be made on these two formulations of the Equivalence Principle (EP). The SEP is valid only in static and homogeneous gravitational field, but it is always possible to choose a sufficiently small space-time region where the gravitational field can be locally approximated by a static homogeneous field, so that the SEP is valid locally. For a scalar particle, the Pauli formulation of EP proposes that a homogeneous gravitational field can always be transformed away globally so that in a suitable reference frame there is only Minkowski space - no gravitational field. On the other hand, Audretsch [34] observes that if one takes a particle with spin, the equation of motion for such a particle will inevitably involve the curvature tensor, which can not be eliminated by any transformation of coordinates. Some authors ignore the influence of curvature (second derivates) or tidal effects, but this means that they get rid of gravitational field. Finally, some results has been obtained for an infinite homogeneous gravitational field (in the entire space) or for an uniformly accelerated boundless reference frame. These gravitational fields are not a true gravitational fields [14].

About the WEP, the equations of motion of a point mass in a curved background spacetime were investigated by Mino, Sasaki and Tanaka [15]. The same equations of motion were later obtained by Quinn and Wald [16, 17] from an axiomatic approach. Following Mino, Quinn and Wald, Haas and Poisson [18] calculate the self-force acting on a point scalar charge in a wide class of cosmological spacetimes. The self-force produce two effects: a time-changing inertial mass and a deviation relative to geodesic motion. The work of Dewitt and Brehme [19], corrected by Hobbs [20], showed that a point charge in true gravitational field not follows a geodesic, so that WEP is violated for a charged particle. Using the techniques of finite-temperature field theory, Donoghue et al. [21] showed that the equality of inertial mass and gravitational mass, for charged spin-$\frac{1}{2}$ or spin-zero particles, is no valid in the context of quantum field theory at finite temperature. Higuchi [35] calculated the position shift of the final-state wave packet of the charged particle due the radiation and showed that it disagrees with the result obtained using the Lorentz-Abraham-Dirac equation for the radiation-reaction force. In an alternative approach, Spohn [36] and other authors [37]-[39] changed the Lorentz-Abraham-Dirac equation for the force on an accelerating charge, which avoids the pathologies of preacceleration and runaway solutions. Yaghjian [40] suggest that these problems will be absent once the finite-size effects are properly taken into account. Finally, the validity of the WEP is very well tested for macroscopic bodies to a sensitivity of few parts in

10^{-13}, but this does not necessarily imply that such principle continues to hold at a microscopic scale and in the quantum regime.

The goal of this work is not to discuss about these papers, but, instead, to add the possibility to analyze the problem of local motion of the classical charged point particles in a different perspective, with emphasis in the EP, which validity is used as a good starting point. As the subject of this work is about the conditions of validity or not of the EP, it is just to notice that results from General Relativity are not used at any moment, in order to avoid any possibility to fall in a vicious causal recurrence.

This text is a review of an old work of Goto [41]. What we have to do is to figure out the condition that we have to provide such that a classical charged point particle can reach a locally stable hyperbolic motion. We show that is necessary to furnish a balance between an applied external force and the electromagnetic radiation reaction force to get an hyperbolic motion regime. An important consequence is that, taking account the SEP, it implies in a passive gravitational mass that is slight greater than the inertial mass. From this result, one show that what seems to be uncomfortable, as the presence of radiation for the charged particle performing hyperbolic motion and its absence for one supported at rest in an uniform gravitational field [42, 43], both equivalent situations as the SEP is valid, lead to a new physical feature performed by charged particles. As consequence of the unexpected behavior of the passive gravitational and inertial mass relation, at a very strong gravitational field, we find the presence of a divergence that indicates a critical field value that signs the validity domain of the SEP.

This paper is organized as follows. In section II, we show that the locally external force necessary to produce an hyperbolic motion in neutral particles is smaller than the locally external force necessary to give the same hyperbolic motion in classical charged particles. In section III, we figure out that, to the SEP to be valid, the WEP is violated for classical charged point particles in stable local hyperbolic motion regime. More, we show that there exists a critical, particle dependent, gravitational field value that signs the validity domain of the SEP. The section IV is devoted to a final discussion and conclusions.

2 Local hyperbolic motion of charged particles

Hyperbolic motion is the natural generalization of the concept of the Newtonian uniformly accelerated motion due to a constant force applied to a particle, which might be due to an uniform gravitational field. At relativistic level, as the velocity is upper limited by the light velocity, constant force don't imply in constant acceleration; instead, it results in the above mentioned hyperbolic motion, which denomination comes from the hyperbola that it is drawn in the zt-plane by this kind of motion.

An one dimensional hyperbolic motion of a particle of mass m occurs as a

solution of the relativistic equation of motion [6, 7]

$$m\frac{d^2x^\mu}{d\tau^2} = f^\mu(\tau), \tag{1}$$

when external force F is parallel to velocity v and it is locally constant in the proper referential frame. In (1) f^μ is the relativistic force defined as

$$f^\mu = \gamma\left(\frac{v\cdot F}{c}, F\right) \qquad \text{with} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \text{and} \quad \beta = v/c\ , \tag{2}$$

where c is the velocity of light.

Supposing the motion along the z-axes, the trajectory of hyperbolic motion is given by

$$(z^0, \mathrm{z}) = \frac{\mathrm{c}^2}{\mathrm{a}}(\sinh\lambda\tau, \cosh\lambda\tau)\ , \tag{3}$$

where $a = F/m$ is a constant proper acceleration and $\lambda = a/c$. From (3) the velocity and acceleration are given by

$$(\dot{z}^0,\ \dot{\mathrm{z}}) = \mathrm{c}(\cosh\lambda\tau, \sinh\lambda\tau) = \gamma\mathrm{c}\,(1,\beta) \tag{4}$$

and

$$(\ddot{z}^0,\ \ddot{\mathrm{z}}) = \mathrm{a}(\sinh\lambda\tau, \cosh\lambda\tau)\ , \tag{5}$$

respectively, so that the relativistic force responsible by the hyperbolic motion is

$$f^\mu(\tau) = m(\ddot{z}^0,\ \ddot{\mathrm{z}}) = \mathrm{ma}(\sinh\lambda\tau, \cosh\lambda\tau)\ . \tag{6}$$

The choice of the metric tensor $g^{\mu\nu}$ is such that $v^\mu v_\mu = -c^2$ for four velocity $v^\mu = \dot{x}^\mu$and, at non relativistic limit, $a^\mu a_\mu = a^2$ for four acceleration $a^\mu = \ddot{x}^\mu$.

The equation of motion of a classical charged point particle, including electromagnetic radiation reaction force, is given by the well known Lorentz-Abraham-Dirac equation [1]-[11], [36]-[39],

$$ma^\mu(\tau) = f^\mu_{ext}(\tau) + f^\mu_{rad}(\tau), \tag{7}$$

where $f^\mu_{ext}(\tau)$ is the external four-force and

$$f^\mu_{rad}(\tau) = m\tau_0\left(\dot{a}^\mu - \frac{1}{c^2}a^\nu a_\nu v^\mu\right), \tag{8}$$

with

$$\tau_0 = \frac{2}{3}\frac{e^2}{mc^3} \ , \tag{9}$$

is the Lorentz-Abraham-Dirac relativistic electromagnetic radiation reaction force. The first term in (8) is known as the Schott term [4] and it is responsible by the well known non-physical runaway solutions. The second is the Rohrlich term, related to the power radiated

$$\mathcal{R} = \frac{d\,\mathrm{W_{rad}}}{dt} = m\tau_0 a^\nu a_\nu \ . \tag{10}$$

A well known condition for hyperbolic motion, which satisfies (3-6), is

$$\dot{a}^\mu - \frac{1}{c^2} a^\nu a_\nu v^\mu = 0 \ , \tag{11}$$

which also implies in $f^\mu_{rad}(\tau) = 0$, so it seems to be easy to produce hyperbolic motion of a charged particle imposing a locally constant external force, as in the uncharged particle case, but it could induce to a misunderstanding. Hyperbolic motion is an ideal concept that imply an eternal constant local acceleration, not existing in a real world, and what happens immediately before reaching this regime will be freezed in the final hyperbolic motion. The important result that we are going to show is that, while the final force that supports the hyperbolic motion for uncharged and charged particles are equal, the composition of such forces is different. For uncharged particles it is just the external force, but for charged particles, it is composed by the sum of external force plus the Rohrlich electromagnetic radiation reaction force. On other words, to have a stable hyperbolic motion for charged particles we have to get a very sensible balance between external and electromagnetic radiation reaction force, and before it the condition (11) is not true. It means that what happen before is very important to get a stable hyperbolic motion regime and, although we have the same Eq.(1) after that, the force $f^\mu(\tau)$ is not just $f^\mu_{ext}(\tau)$ anymore. To figure out why, let us consider the Lorentz-Abraham-Dirac Eq.(7) written as [11]

$$m(1 - \tau_0 \frac{d}{d\tau}) a^\mu = f^\mu_{ext}(\tau) - \frac{1}{c^2}\mathcal{R} v^\mu = K^\mu(\tau) \ . \tag{12}$$

Formal expansion like

$$(1 - \tau_0 \frac{d}{d\tau})^{-1} = 1 + \tau_0 \frac{d}{d\tau} + \tau_0^2 \frac{d^2}{d\tau^2} + \cdots \tag{13}$$

enables us to get a formal solution of Lorentz-Abraham-Dirac equation as

$$m a^\mu(\tau) = \sum_{n=0}^{\infty} \tau_0^n \frac{d^n}{d\tau^n} K^\mu(\tau) \ . \tag{14}$$

We can insert the mathematical identity

$$\frac{1}{n!}\int_0^\infty s^n e^{-s} ds = 1 \qquad (15)$$

to transform (14) in a second order integro-differential equation

$$ma^\mu(\tau) = \int_0^\infty e^{-s} K^\mu(\tau + \tau_0 s) ds \ , \qquad (16)$$

which shows a possible non-causal behavior. In an explicit form, we have

$$ma^\mu(\tau) = \int_0^\infty \left. (f^\mu_{ext} - \frac{1}{c^2}\mathcal{R} v^\mu)\right|_{\tau+\tau_0 s} e^{-s} ds \ . \qquad (17)$$

The Eq.(7), or the equivalent Eq.(17), are valid during the hyperbolic motion. On the other hand, while the motion is approaching the hyperbolic regime, as discussed above, we have the limiting process

$$\dot{a}^\mu - \frac{1}{c^2} a^\nu a_\nu v^\mu \to 0 \Rightarrow f^\mu_{rad}(\tau) \to 0 \ , \qquad (18)$$

such that the total force behaves like

$$f^\mu_{ext}(\tau) + f^\mu_{rad}(\tau) \to f^\mu(\tau) \ , \qquad (19)$$

or, $f^\mu_{ext} \to f^\mu$, so that the Eq.(17), in this regime, can be rewritten as

$$\int_0^\infty \left. (f^\mu - \frac{1}{c^2}\mathcal{R} v^\mu)\right|_{\tau+\tau_0 s} e^{-s} ds = f^\mu(\tau) \ , \qquad (20)$$

recovering the Eq.(1), in accordance with Eq.(7) and condition (11).

From (20), to have an hyperbolic motion, it is necessary that the applied external force goes to

$$f^\mu_{ext}(\tau) \to f^\mu_{ext}(\tau) = \int_0^\infty f^\mu(\tau + \tau_0 s) e^{-s} ds \ , \qquad (21)$$

as well as the electromagnetic radiation reaction force goes to

$$f^\mu_{rad}(\tau) \to f^\mu_{Roh}(\tau) = \int_0^\infty \left. \frac{1}{c^2}\mathcal{R} v^\mu \right|_{\tau+\tau_0 s} e^{-s} ds \ , \qquad (22)$$

such that, from (19-22),

$$f^\mu(\tau) = f^\mu_{ext}(\tau) + f^\mu_{Roh}(\tau) \ , \qquad (23)$$

where $f^\mu(\tau)$ is given by (6).

Using (6), the spatial component of Eq.(21) becomes

$$f_{ext}(\tau) = \frac{ma}{2}\left(\frac{e^{\lambda\tau}}{(1-\lambda\tau_0)} + \frac{e^{-\lambda\tau}}{(1+\lambda\tau_0)}\right) \ . \tag{24}$$

Analogously, from Eqs.(4-6) and (10), the spatial component of Eq.(22) becomes

$$f_{Roh}(\tau) = -\frac{ma}{2}\lambda\tau_0\left(\frac{e^{\lambda\tau}}{(1-\lambda\tau_0)} - \frac{e^{-\lambda\tau}}{(1+\lambda\tau_0)}\right) \ . \tag{25}$$

In the same way, time components become

$$f^0_{ext}(\tau) = \frac{ma}{2}\left(\frac{e^{\lambda\tau}}{(1-\lambda\tau_0)} - \frac{e^{-\lambda\tau}}{(1+\lambda\tau_0)}\right) \tag{26}$$

and

$$f^0_{Roh}(\tau) = -\frac{ma}{2}\lambda\tau_0\left(\frac{e^{\lambda\tau}}{(1-\lambda\tau_0)} + \frac{e^{-\lambda\tau}}{(1+\lambda\tau_0)}\right) \ , \tag{27}$$

where $(1-\lambda\tau_0) > 0$ in (24-27). For $(1-\lambda\tau_0) < 0$ the integrals (21-22) are divergent. In section 3, this condition will be discussed.

From Eqs.(24-27), it is easy to see that the total force (23) satisfies (6), condition necessary to have an hyperbolic motion. It shows that the external force necessary to produce an hyperbolic motion in neutral particles,

$$f^{(neutral)}_{ext}(\tau) = ma\cosh\lambda\tau \ , \tag{28}$$

is smaller than the external force (24) necessary to give the same hyperbolic motion in charged particles. All external force applied to a neutral particle is used to increase its kinetic energy,

$$\frac{d\,\mathrm{W}}{dt} = v\ F = m\ a\ v = m\ a\ c\ \sinh\lambda\tau = m\ c^2\ \frac{d\,\gamma}{d\tau} \ , \tag{29}$$

where $\gamma = \cosh\lambda\tau$ from (4) and $\lambda = a/c$. On the other hand, for charged particles, the external force (24), that can be written as

$$f^{(charged)}_{ext}(\tau) = f^{(neutral)}_{ext}(\tau) - f_{Roh}(\tau) \ , \tag{30}$$

provides the increase of kinetic energy in the same amount as given in (29) and supplies, through $f_{Roh}(\tau)$, the energy lost carried by electromagnetic radiation.

3 Classical charged particles in a local uniform gravitational field

We saw in the previous section that a classical charged particle performing hyperbolic motion have to be submitted to an external force given by

$$F_{ext} = \frac{ma}{2}\left(\frac{(1+\beta)}{(1-\lambda\tau_0)} + \frac{(1-\beta)}{(1+\lambda\tau_0)}\right), \tag{31}$$

where F_{ext} is the spatial component of the measurable force related to the relativistic force by $f_{ext}(\tau) = \gamma F_{ext}(\tau)$ [see (2)]. To obtain (31) observe, from (4), that $\gamma(1+\beta) = e^{\lambda\tau}$ and $\gamma(1-\beta) = e^{-\lambda\tau}$.

The SEP [10, 44] says that a particle at rest in the laboratory frame R_{lab} immersed in an uniform gravitational field g is seen by observer in a free falling inertial frame R_{in} as performing hyperbolic motion with local constant acceleration $a = g$. The local constant force responsible by its hyperbolic motion is the normal force $F_n = -F_g$ that supports the particle against the gravitational force F_g, so that, in absolute value it is equal to mg for a uncharged particle. But, for a charged particle, the normal force F_n must be equal to the external force F_{ext} of Eq.(31) for $\beta = 0$,

$$F_{ext} \to F_n = \frac{mg}{(1-\lambda^2\tau_0^2)}\ . \tag{32}$$

This result suggests that the observer in the laboratory frame R_{lab} measures the gravitational force acting on a charged particle as

$$F_g = -\frac{mg}{(1-\lambda^2\tau_0^2)} \tag{33}$$

such that

$$m^* = \frac{m}{(1-\lambda^2\tau_0^2)} \tag{34}$$

should define the passive gravitational mass m^* of a charged particle with inertial mass m. The relation (34) shows that, to the SEP to be valid, the WEP is violated for classical charged particles in stable hyperbolic motion regime.

On other words, the external force applied on uncharged particles must be slightly lower than for charged particles. The first one disposes all the external force to increase its kinetic energy, while the charged one needs an external force to supply the same kinetic energy plus the energy lost by electromagnetic radiation. As the strong version of the EP is invoked, the equivalence between the proper uniformly accelerated hyperbolic motion referential and the referential supported in the presence of an uniform gravitational field implies that the

charged particle gravitational force, as well as the gravitational potential energy, must be slightly greater than for the uncharged one. This difference should be interpreted as due to their different gravitational masses and the same inertial masses. It implies that the gravitational and inertial masses of charged particles are different.

For typical charged particles, as electron or proton, $\tau_0 \simeq 6.3 \times 10^{-24}s$ and $\tau_0 \simeq 3.4 \times 10^{-27}s$, respectively. In a field magnitude typical for a terrestrial gravitational field $g \simeq 10\ m/s^2$, we have $\lambda = g/c \simeq 3.3 \times 10^{-8}s^{-1}$. So, we can see that $\lambda^2\tau_0^2 \simeq 4.\,3 \times 10^{-62}$ and $\lambda^2\tau_0^2 \simeq 1.3 \times 10^{-68}$, respectively for electron and proton, very small numbers, such that $1 - \lambda^2\tau_0^2 \cong 1$. As a consequence, the passive gravitational mass is just slight greater than the inertial mass, $m^* \gtrsim m$, and the gravitational and inertial mass relation defined by Eq.(34) is much as close to unit, $r = m^*/m \cong 1$. It means that there is no consequence, for practical purpose, due to this slight up deviation of passive gravitational mass in relation to the inertial mass, at least in a region with gravitational field of magnitude as considered above. In fact, it is true for a very long field interval, starting with $g = 0$ and going until to reach a very strong gravitational field of the order $g \sim 10^{30}m/s^2$.

In a such strong gravitational field region, the field dependence of $r = m^*/m$, see Eq.(34), starts to manifest, as we can see in the figure 1. It increases very slowly and remains very close to unit, starting with $g = 0$ until to reach the very strong field magnitude of the order $g \sim 10^{30}m/s^2$, approaching the divergence point given by the condition $1 - \lambda^2\tau_0^2 = 0$. Then, r increases fast to the infinite as the gravitational field goes to its critical value $g_c = c/\tau_0$. This critical field value is mass dependent, with $g_c \simeq 4.8 \times 10^{31}m/s^2$ and $g_c \simeq 8.8 \times 10^{34}m/s^2$ for electron and proton, respectively.

The divergence of the gravitational and inertial mass relation at the critical field value signs that the SEP should not be valid in this situation. Consistently, the condition $(1 - \lambda\tau_0) > 0$ for which the integrals (21-22) are finites, implies that $(1 - \lambda\tau_0)(1 + \lambda\tau_0) = (1 - \lambda^2\tau_0^2) > 0$, so that the investigation of the values above the critical point g_c, as the mass relation r turns to be negative, is nonsense.

Where is possible to find such strong gravitational field? Astrophysical compact objects are the natural place to search for more intense gravitational fields. For example, white dwarf like Sirius B, with mass close to the solar mass, $1.0 \times M_\odot$, and radius near $5.5 \times 10^6 m \simeq 0.008R_\odot$, is a very compact object with a surface gravitational acceleration $g \simeq 4.6 \times 10^6 m/s^2$ [45]. (The solar mass and radius are $M_\odot \simeq 1.98844 \times 10^{30}kg$ and $R_\odot \simeq 6.961 \times 10^8 m$, respectively).

Neutron stars are more compact than white dwarfs. For mass of order $1.4 \times M_\odot$ corresponds a radius of order $R \simeq 4.4 \times 10^3 m$ with a surface gravitational acceleration $g \simeq 2 \times 10^{12}m/s^2$ [46].

Extreme compact objects are of course the black holes. Caution is need to deal with such objects, because there is no back information access beyond their events horizon, defined by the spherical surface at the Schwarzschild radius

given by [46]

$$R_S = \frac{2MG}{c^2} \simeq 2.95 \times \frac{M}{M_\odot} km.$$

The gravitational acceleration at the Schwarzschild surface is [46]

$$g = \frac{MG}{R_S^2} = \frac{c^4}{4MG} \simeq 1.5 \times 10^{13} \frac{M_\odot}{M} m/s^2.$$

For a typical black hole with about ten times solar mass, $M \simeq 10 \times M_\odot$, $R_S \simeq 30km$ and the gravitational acceleration at its Schwarzschild surface is $g \simeq 1.5 \times 10^{12} m/s^2$. It is belief that galaxies central region shelter very massive black holes, with masses of order $10^5 \times M_\odot$ to $10^9 \times M_\odot$. Such black hole, with mass $M \simeq 10^5 \times M_\odot$ has $R_S \simeq 2.95 \times 10^5 km$ and gravitational acceleration at its Schwarzschild surface $g \simeq 1.5 \times 10^8 m/s^2$. If the mass is $M \simeq 10^9 \times M_\odot$, the Schwarzschild radius is $R_S \simeq 2.95 \times 10^9 km$ and $g \simeq 1.5 \times 10^4 m/s^2$. Increasing the black hole mass doesn't imply increasing the gravitational acceleration, because the Schwarzschild radius increase together. It shows that astrophysical environment hardly get us gravitational acceleration stronger than about $g \sim 10^{13} m/s^2$, which implies $\lambda^2 \tau_0^2 \sim 10^{-36}$, nothing to worry about. Only gravitational acceleration strong as $g \sim 10^{30} m/s^2$ are going to be sensible in equation (34), close to an infinite singularity. Such too strong gravitational field is very unlikely.

On the other hand, collision of very energetic particles could take place with instantaneous acceleration that might be of such order, simulating a local and instantly gravitation, as that needed to test the equivalence principle. Nevertheless, it seems to be more comfortable to abdicate of the Principle of Equivalence in such extreme regime, where quantum effects certainly are dominant, and the gravitation, as it is a belief, is going to be unified with the other interactions.

4 Conclusion

Although the final equation of motion of classical neutral and charged particles in hyperbolic motion regime seems to be identical, we realize that there is a fundamental difference between them. For classical charged particles, actually, the total locally constant force is the sum of an applied and the electromagnetic radiation reaction forces in a combination in such a way that results the same as for neutral particles. An interesting implication, as the SEP is taken account, is that the passive gravitational mass of the charged particle must be greater than its inertial mass in a very small amount. It is small enough to not be detected by any experimental or practical devices, but it helps us to figure out the condition necessary to get an hyperbolic motion regime and to understand

the meaning of its equivalence with a charged particle supported at rest in an uniform gravitational field.

Until now the equality between gravitational and inertial mass was understood as the essence of the EP, condition that we realized not to be true for charged particles, since, due to the presence of electromagnetic radiation reaction, a slight deviation of gravitational mass compared to inertial one is necessary to hold the SEP. However, a new feature comes from the gravitational and inertial mass relation behavior for a very strong gravitational field. There exists a critical, particle dependent, gravitational field value that signs the validity domain of the SEP. For electron and proton these critical field values are $g_c \simeq 4.8 \times 10^{31} m/s^2$ and $g_c \simeq 8.8 \times 10^{34} m/s^2$, respectively. They possibly coincide where the quantum effects turns to be relevant.

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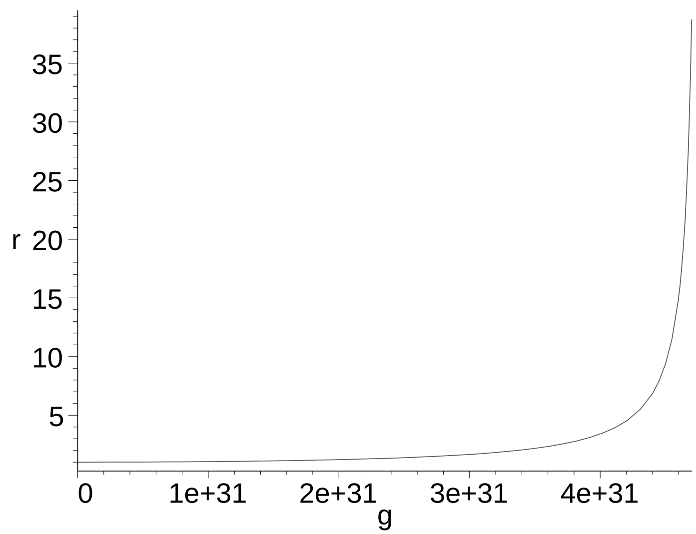
Figure Caption

Figure 1 - Electron gravitational and inertial mass relation, $r = m^*/m$, as function of the gravitational field g. There is a critical point defined by $g_c = c/\tau_0$, which is particle dependent, and has the value $g_c \simeq 4.8 \times 10^{31} m/s^2$ for electron.



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5
0
1e+31
2e+31
3e+31
4e+31
g